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Question Paper Code : 70769

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2021.

Fourth Semester

Electronics and Communication Engineering

MA 6451 – PROBABILITY AND RANDOM PROCESSES

(Common to Biomedical Engineering, Robotics and Automation Engineering)

(Regulations 2013)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Let X be a random variable and $P(X=x)$ is the probability mass function given by

x:	0	1	2	3	4	5	6	7
$P(X=x)$	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

Determine the value of k.

2. Find the mean of the random variable X whose moment generating function is given by $\frac{1}{81}(2t+1)^4$.

3. The joint probability mass function of a two dimensional random variable (X, Y) is given by $P(x,\ y)=K(2x+y), x=1,2$ and y = 1, 2 where K is a constant. Find the value of K.

4. The two regression equations of two random variables X and Y are $4x-5y+33=0$ and $20x-9y=107$. Find the mean values of X and Y.

5. Define wide sense stationary process.

6. If the customers arrive at a bank according to a Poisson process with mean rate 2 per minute, find the probability that during a 1-minute interval no customer arrives.

7. The Power Spectral density of a random process *x(t)* is given by

$$S_{xx}(W) = \begin{cases} \pi & if|W| < 1 \\ 0 & elsewhere \end{cases}$$

Find its autocorrelation function.

8. Prove that $R_{xy}(\tau) = R_{yx}(-\tau)$

9. Prove that $Y(t) = 2X(t)$ is linear.

10. State the relation between input and output of a linear time invariant system.

PART B — (5 × 16 = 80 marks)

11. (a) (i) A car hire firm has 2 cars. The number of demands for a car on each day is distributed as Poisson variate with mean 0.5. Calculate the proportion of days on which

(1) Neither car is used

(2) Some demand is refused. (8)

(ii) Suppose X has an exponential distribution with mean equal to 10. Determine the value of x such that $P(X < x) = 0.95$. (8)

Or

(b) (i) A random variable Y is defined as $\cos \pi x$, where X has a uniform probability density function over $\left(-\frac{1}{2}, -\frac{1}{2}\right)$. Find the mean and standard deviation. (8)

(ii) A manufacturer produces covers where weight is normal with mean $\mu = 1.950g$ and S.D. $\sigma = 0.025g$. The covers are sold in lots of 1000. How many covers in a lot may be heavier than 2 g? (8)

12. (a) (i) Find the marginal density functions of X and Y if the joint probability density function is

$$f(x, y) = \begin{cases} \frac{2}{5}(2x + 3y); & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0 & ;\ otherwise \end{cases}$$ (8)

(ii) Calculate the coefficient of correlation between the variables X and Y from the data given below. (8)

X	64	65	66	67	68	69	70
Y	66	67	65	68	70	68	72

Or

(b) (i) If X and Y each follow an exponential distribution with parameter 1 and are independent, find the probability density function of $U = X - Y$. (8)

(ii) Find $E(XY)$ if the joint probability density function of two dimensional random variables (X,Y) is given by

$$f(x, y) = 24y(1 - x) : 0 \le y \le x \le 1. \quad (8)$$

13. (a) (i) Show that the process $X(t) = A\cos\lambda t + B\sin\lambda t$ where A and B are random variables, is wide sense stationary process if

$$E(A) = E(B) = E(AB) = 0,\ E(A^2) = E(B^2). \quad (8)$$

(ii) There are 2 white marbles in Urn A and 3 red marbles in Urn B. At each step of the process, a marble is selected from each urn and the 2 marbles selected are interchanged. The state of the related Markov chain is the number of red marbles in Urn A after the interchange. What is the probability that there are 2 red marbles in Urn A after 3 steps? In the long run, what is the probability that there are 2 red marbles in Urn A? (8)

Or

(b) (i) A radioactive source emits particles at a rate of 5 per minute in accordance with Poisson process. Each particle emitted has a probability 0.6 of being recorded. Find the probability that 10 particles are recorded in 4 minute period. (8)

(ii) Check if a random telegraph signal process is wide sense stationary. (8)

14. (a) (i) The autocorrelation function of an ergodic process $x(t)$ is

$$R_{xx}(t) = \begin{cases} 1 - |\tau|, if\ |\tau| \le 1 \\ 0 \quad , otherwise \end{cases}$$

Obtain the spectral density of $x(t)$. (8)

(ii) The cross power spectrum of real random processes $x(t)$ and $y(t)$ is given $S_{xy}(W) = \begin{cases} a + ibw \quad , if\ |w| < 1 \\ 0 \quad , elsewhere \end{cases}$

Find the cross correlation function (8)

Or

(b) (i) If {x(t)} and {y(t)} are two random processes with autocorrelation functions $R_{xx}(\tau)$ and $R_{yy}(\tau)$ respectively and jointly WSS, then prove that $|R_{xy}(\tau)| \leq \sqrt{R_{xx}(0) - R_{yy}(0)}$. Establish any two properties of autocorrelation function $R_{xx}(\tau)$. (8)

(ii) Given the power spectral density of a continuous process as $S_{xx}(\mathrm{w}) = \frac{\mathrm{w}^2 + 9}{\mathrm{w}^4 + 5\mathrm{w}^2 + 4}$. Find the mean square value of the process. (8)

15. (a) If {$X(t)$} is a WSS process and if $Y(t) = \int_{-\infty}^{\infty} h(u) \times (t - u)\, du$, prove that:

(i) $R_{XY}(\tau) = R_{XX}(\tau) * h(-\tau)$ and

(ii) $R_{YY}(\tau) = R_{XY}(\tau) * h(\tau)$ where * denotes convolution

(iii) $S_{XY}(\omega) = S_{XX}(\omega) H^*(\omega)$ where $H^*(\omega)$ is the complex conjugate of $H(\omega)$

(iv) $S_{YY}(\omega) = S_{XX}(\omega) |H(\omega)|^2$ (16)

Or

(b) A Random process $X(t)$ is the input to a linear system whose impulse response is $h(t) = 2e^{-t}$, $t \geq 0$. If the autocorrelation function of the process is $R_{XX}(\tau) = e^{-2|\tau|}$, determine the cross correlation function $R_{XY}(\tau)$ between the input process $X(t)$ and the output process $Y(t)$ and the cross correlation function $R_{YX}(\tau)$ between the output process $Y(t)$ and the input process $X(t)$. (16)

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